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Universal Tracking Controller with Disturbance Rejection

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***Abstract*—In this paper, an extremely simple yet super effective universal tracking controller (UTC) is developed based on integral chain system, which can achieve accurate tracking for continuous signals as well as superior rejection for disturbances regardless of system models. We discovered two natural formulations for UTC: one is the proportional-integral tracking controller (PITC), which includes a proportional and an integral part of states errors; the other is the adaptive feedforward tracking controller (AFTC), which consists of a feedback part of state errors and a feedforward part which is obtained adaptively by using the previous sampled input. When the integral gain is high, it is found that PITC and AFTC are approximately equivalent with the integral gain be proportional to the sampling rate. Thus PITC and AFTC together form a unified framework of UTC, where PI and PID are only particular cases of first-order and second-order PITC, respectively.**

I. Introduction

Proportional-integral-derivative (PID) control [1-2] is the most widely used control strategy today. It is estimated that over 90% of control loops are using PID control, most of which with the derivative gain set to zero (PI control). The studies of PID control are mostly based on classical control theory, e.g., the transfer function. As modern control theory appears, most attentions are attracted to the study of nonlinear control, adaptive control and robust control, etc. Over the last half-century, though a great deal of academic and industrial effort has been put on PID control, they are primarily in the areas of tuning rules, identification schemes, and adaptation techniques. As a result, few paper have studied the PID control theory itself in a modern way and the main framework of PID control nearly stays the same as it came into being. With the development of modern control theory, we think it is appropriate time to "rediscover" and develop PID control theory in the modern control framework.

For PID control, a common formulation is usually written as follows

$$u = k_p e + k_i \int_0^t e(\tau)\, d\tau + k_d \dot{e} \tag{1}$$

where the input is a combination of three components about the output tracking error e, i.e., the proportional part, the integral part, and the derivative part.

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Though PID control has been proved very effective for set-point control which can achieve zero steady-state error, it is not considered as a good choice to track time-varying signals. However, we find that with minor modification, the traditional PID control can be turned into a universal tracking controller (UTC), which is much more powerful in tracking time-varying signal and rejecting disturbance. Our inspiration comes from modern control theory. In modern control theory, the system is described in a state space form, and thus using full states feedback is quite common. Especially in nonlinear control, the output tracking error and its high-order derivatives are employed to realize asymptotical tracking. Inspired by these, we realize that only using output error feedback is not enough and may limit the performance of PID control. Therefore we try to modify PID control by replacing the output tracking error by tracking errors of all states and find something surprising. We find it can achieve accurate tracking for continuous signals as well superior rejection for disturbances when a relatively high integral gain is set. In a sense, it is a kind of universal tracking controller since it doesn't depend on the system model. We call it proportional-integral tracking controller (PITC) since it inherits from PI control. PITC is a generalization of PID control and can be applied to multi-input-multi-output (MIMO) higher-order systems, where PI control and PID control are only particular cases of first-order and second-order PITC, respectively.

In addition, from the perspective of tracking control theory, we find another natural formulation of UTC. Tracking control has been studied in some modern control branches, e.g. the output regulation theory [3-4], and inversion control theory [5-7]. Output regulation [3-4] is to control a fixed plant in order to have its output track (or reject) a family of reference (or disturbance) signals produced by some external generator. The core of output regulation is the internal model principle [8]: A controller which incorporates an internal model of the exosystem is able to secure asymptotic decay to zero of the tracking error for every possible exogenous input in the class of signals generated by the exosystem. While the inversion control theory tries to find out an ideal input which renders exact tracking by making inversion of system dynamics online (e.g. dynamic inversion [5]) or offline (e.g. stable inversion [6-7]).

Among these two control theories, a quite often used controller is a feedback and feedforward controller:

$$u = u_f + \mathbf{K}\left(\mathbf{x}_\mathrm{r} - \mathbf{x}\right) \tag{2}$$

where u_f is the ideal input and serves as a feedforward control part and $\mathbf{K}(\mathbf{x_r}-\mathbf{x})$ is a feedback control part with $\mathbf{K}$ being the feedback control gain and $\mathbf{x_r}$ being the state references. This control structure is quite regular in output regulation theory [3-4] and inversion control theory [6-7]. It can be interpreted as: the feedforward part u_f provides the needed control input for exact output tracking, while the feedback part $\mathbf{K}(\mathbf{x_r}-\mathbf{x})$ gives a back force when the system states are deviated from the states references and thus keeps the system stable. Though this is a linear control law, it makes the system locally exponentially stable at the desired trajectory.

However, both output regulation theory and inversion control theory depend on system model to find out the feedforward part. In output regulation theory, the feedforward part is obtained by solving the regulation equation; in inversion control theory, the feedforward part is obtained from model inversion. Thus both of them are sensitive to model uncertainties or disturbances. Inspired by the idea in iterative learning control [9] and policy iteration [10], we find out a model-free way to figure out the ideal input. The feedforward is obtained iteratively by using the input in previous time. At initial time, the feedforward is set as zero. Each time the whole input is updated by the feedback part due to the state errors, and is used as feedforward at the next time. As time goes, the feedforward part adaptively converges to a small region around the ideal input. Since the feedforward part is obtained adaptively, we call it adaptive feedforward tracking controller (AFTC). Like PITC, AFTC can also achieve accurate tracking for continuous signals and excellent rejection for disturbances.

Thus starting from two different directions we have built two kinds of UTC: PITC comes from PID while AFTC is derived from tracking control theory. Frankly speaking, AFTC is more intrinsic since it gives us direct understanding of its universal tracking ability. However, with further exploration, we find that they are actually equivalent to each other with the integral gain in PITC be proportional to the sampling rate in AFTC. Thus they are just two formulations of UTC.

The main contribution of this paper is the proposition of UTC, which is an extension of PID control in the state space. UTC gives us an in-depth understanding of PID control and the tracking control theory. Due to the universality and excellent performance, UTC may have broad applications in the future.

The rest of this paper is organized as follows. Section II gives the main results of UTC and Section III are simulations and discussions. Section IV points out some open problems and Section V concludes this paper.

II. The Main Results of Universal Tracking Controller

The system considered is a multi-input-multi-output (MIMO) nonlinear system with m inputs and m outputs. Denote $\mathbf{u}=[u_1,u_2,\ldots,u_m]^T$ as the inputs vector, $\mathbf{y}=[y_1,y_2,\ldots,y_m]^T$ as the outputs vector, $\{r_1,r_2,\ldots,r_m\}$ be the relative degree which represents the order of differentiations when the input first appears in the output derivatives. Suppose there are no zero dynamics in this system (equals to no transmission zeros for linear system), then the system can be written in an integral chain form:

$$\begin{aligned} y_1^{(r_1)} &= f_1(\mathbf{x},\mathbf{u})+d_1 \\ y_2^{(r_2)} &= f_2(\mathbf{x},\mathbf{u})+d_2 \\ &\ldots \\ y_m^{(r_m)} &= f_m(\mathbf{x},\mathbf{u})+d_m \end{aligned} \tag{3}$$

where $\mathbf{x}=\left[y_1,\dot{y}_1,\ldots,y_1^{(r_1-1)},\ldots,y_m,\dot{y}_m,\ldots,y_m^{(r_m-1)}\right]^T\in\mathbb{R}^n$ is the state vector, and $\mathbf{d}=[d_1,d_2,\ldots,d_m]^T$ represents some external disturbances. The control objective is to let the output $\mathbf{y}=[y_1,y_2,\ldots,y_m]^T$ track some continuous references $\mathbf{y_r}(t)=\left[y_{1r}(t),y_{2r}(t),\ldots,y_{mr}(t)\right]^T$. It should be noted that this model is only used for description. The exact model is not necessarily known and will not be used in controller design.

Remark 1: A system which can be written in the form of (3) is also called a flat system [11], meaning that we can find a set of outputs such that all states can be determined from these outputs without integration.

For this system, it has a very good property that if the output references $\mathbf{y_r}(t)=\left[y_{1r}(t),y_{2r}(t),\ldots,y_{mr}(t)\right]^T$ are given, then all the states references are known, i.e., $\mathbf{x_r}=\left[y_{1r},\dot{y}_{1r},\ldots,y_{1r}^{(r_1-1)},\ldots,y_{mr},\dot{y}_{mr},\ldots,y_{mr}^{(r_m-1)}\right]^T$. Thus all the states tracking errors $\mathbf{e_x}=\mathbf{x}-\mathbf{x_r}$ are available for feedback instead of only using output error feedback.

A. Proportional-Integral Tracking Controller

The control architecture of PITC is shown in Fig. 1.

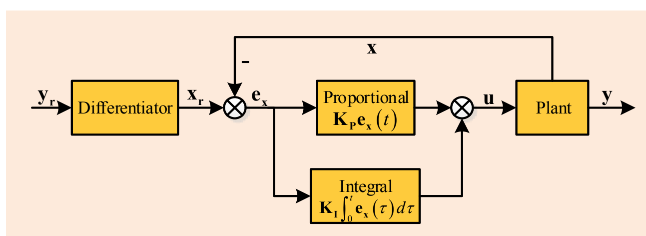

Fig. 1. Control Architecture of PITC.

PITC consists of a proportional part and an integral part of states errors:

$$\mathbf{u}=\mathbf{K_P}(\mathbf{x_r}-\mathbf{x})+\mathbf{K_I}\int_0^t(\mathbf{x_r}-\mathbf{x})d\tau \tag{4}$$

where $\mathbf{K_P}\in\mathbb{R}^{m\times n},\mathbf{K_I}\in\mathbb{R}^{m\times n}$ are the proportional and integral gain matrices, respectively. For a SISO system, PITC becomes

$$u = k_{p1}\left(x_r - x\right) + k_{p2}\left(\dot{x}_r - \dot{x}\right) + \ldots + k_{pn}\left(x_r^{(n-1)} - x^{(n-1)}\right) + k_{i1}\int_0^t \left(x_r - x\right)d\tau + k_{i2}\int_0^t \left(\dot{x}_r - \dot{x}\right)d\tau + \ldots + k_{in}\int_0^t \left(x_r^{(n-1)} - x^{(n-1)}\right)d\tau \tag{5}$$

From (5) it can be seen the difference between PITC and PI/PID controller: PITC uses not only output error but also the higher-order derivatives of output error as feedback, while PI controller uses only output error as feedback and PID uses output error and its first-order derivative as feedback. Therefore, PITC can be seen as a generalization of PI/PID control.

Since PITC uses high-order derivatives of the output, it can work together with some differentiator when only the output is measurable. However, some actions will need to be taken considering the measurement noises.

B. *Adaptive Feedforward Tracking Controller*

The control architecture of AFTC is shown in Fig. 2.

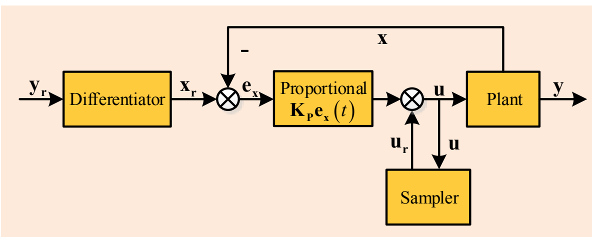

Fig. 2. Control Architecture of AFTC.

AFTC consists of a feedback part of state errors and a feedforward part which is the previous input with a fixed time delay T:

$$\begin{aligned} \mathbf{u} &= \mathbf{u_r} + \mathbf{K_P}\left(\mathbf{x_r} - \mathbf{x}\right) \\ \mathbf{u_r}(t) &= \mathbf{u}(t - T) \end{aligned} \tag{6}$$

In reality, the previous input is not recorded in continuous time but in some sampling time, thus we can use the latest sampling input as the feedforward for the current time. Therefore, a realistic formulation of AFTC can be written as

$$\begin{aligned} \mathbf{u} &= \mathbf{u_r} + \mathbf{K_P}\left(\mathbf{x_r} - \mathbf{x}\right) \\ \mathbf{u_r}(t) &= \mathbf{u}(t_n), t_n < t \le t_n + T \end{aligned} \tag{7}$$

In this controller, the feedforward part and the feedback part work together to achieve accurate tracking. The effectiveness of the controller is based on the continuity of the ideal input. If the sampling time is small enough, we can use the previous input to approximate the current ideal input since the ideal input changes a little bit within a short time. At the initial time, the feedforward input is set to zero, which may be biased from the ideal input and will result in big initial tracking error. Then, the feedback part works and produces a big control input. The whole input which is closer to the ideal input is used as feedforward in the next time. As a result, the feedforward keep updating towards the ideal input. Finally, a dynamic equilibrium is approached: the state error stays in a small region around zero which is enough to compensate the small difference between the current feedforward and the next ideal input. In a word, the function of feedforward is holding, i.e., to maintain the current input around the previous input; while the function of feedback is correction, i.e. to update the current input towards the ideal input.

C. *Equivalence between PITC and AFTC*

To explore the relationship between PITC and AFTC, take the first order SISO system as an example. The corresponding PITC is

$$u = k_i \int_0^t e(\tau)d\tau + k_p e(t) \tag{8}$$

And the corresponding AFTC is

$$\begin{aligned} u &= u_r(t) + k_p e(t) \\ u_r(t) &= u(t_n), t_n < t \le t_n + T \end{aligned} \tag{9}$$

To be clear, a diagram of the tracking error is shown in Fig. 3.

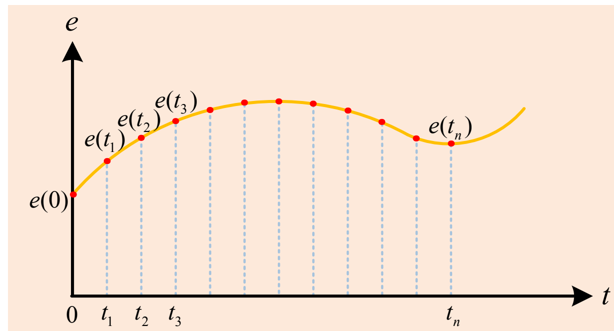

Fig. 3. Diagram of the Tracking Error.

where $t_1 = T, t_2 = 2T, \ldots, t_n = nT$ are the sampling time instants, and the current time t satisfies $t_n < t \le t_n + T$.

For the AFTC (9), by starting from the time origin, the feedforward input is obtained iteratively as

$$\begin{aligned} & u_r(0) = 0, u(0) = k_p e(0) \\ & u_r(t_1) = k_p e(0), \mathrm{u}(t_1) = k_p e(0) + k_p e(t_1) \\ & \ldots \\ & u_r(t_n) = k_p e(0) + k_p e(t_1) + \ldots + k_p e(t_{n-1}) \\ & u_r(t) = k_p e(0) + k_p e(t_1) + \ldots + k_p e(t_{n-1}) + k_p e(t_n) \end{aligned} \tag{10}$$

Thus the current control input is obtained as

$$u(t) = k_p\left[e(0) + e(t_1) + \ldots + e(t_n)\right] + k_p e(t) \tag{11}$$

It can be observed that the feedforward part in AFTC is actually a summation of all the sampling inputs. While in PITC, the integral part represents the area under the tracking error curve. Since the sampling time T is small, the integral part can be approximated by

$$\int_0^t e(\tau)d\tau \approx \left[e(0) + e(t_1) + \ldots + e(t_n)\right]T \tag{12}$$

Thus PITC is approximated by

$$u(t) \approx k_i\left[e(0) + e(t_1) + \ldots + e(t_{n-1})\right]T + k_p e(t)dt \tag{13}$$

By comparing (11) and (13) we can easily draw the conclusion that PITC approximately equals to AFTC if we select $k_i = k_p / T$. Therefore the two kinds of UTC are actually equivalent to each other with the integral gain in PITC be proportional to the sampling rate in AFTC. Generally speaking, AFTC with smaller sampling time corresponds to PITC with bigger integral gain. Since the sampling time T should be small to ensure a good tracking, this can explain why the integral gain in PITC should be big.

With this relationship, a unified framework of UTC can be drawn as shown in Fig. 4.

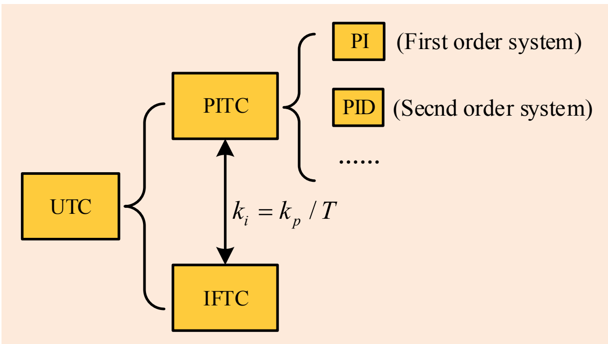

Fig. 4. A Unified Framework of UTC.

This also gives us another perspective to view the three components in PID control, which is quite different from our common sense. The integral part which uses the past information actually plays the role of feedforward, which is a kind of prediction; while the derivative part, which used to be thought as a prediction, is an ordinary state feedback which has the same role as the proportional part in UTC.

III. SIMULATIONS AND DISCUSSIONS

Consider a nonlinear system as follows

$$\begin{aligned} \dot{x}_1 &= x_1^2 + x_2 + \sin(x_1) + 1 + (x_1^2+1)u_1 + d_1 \\ \dddot{x}_2 &= \ddot{x}_2^2 + 1/(1+\dot{x}_2^2) + x_1 x_2 + u_1 + u_2 + d_2 \end{aligned} \tag{14}$$

For this system, the outputs are $\mathbf{y} = [x_1, x_2]^T$ and the states vector is $\mathbf{x} = [x_1, x_2, \dot{x}_2, \ddot{x}_2]^T$. The disturbance is set as a sinusoidal disturbance with time-varying frequency(see Fig. 5).

$$d_1 = d_2 = \sin\left(\frac{50+t}{20+2t}t\right) \tag{15}$$

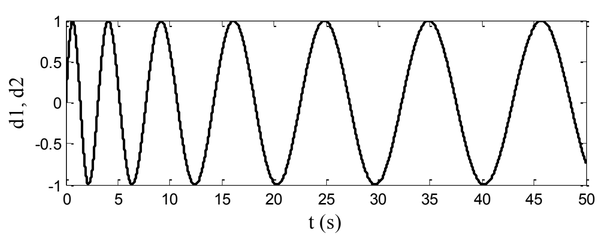

Fig. 5. Sinusoidal Disturbance with Time-varying Frequency.

The outputs references are selected as a sinusoidal signal and a piecewise continuous signal, respectively. The feedback gain is set as $\mathbf{K_P} = [10\ \ 2\ \ 1\ \ 0; 1\ \ 10\ \ 10\ \ 5]$ for both PITC and AFTC. The sampling time for AFTC is selected as $T = 0.01s$ and the integral gain for PITC is set as $\mathbf{K_I} = 100\mathbf{K_P}$. The simulation results of PITC and AFTC are nearly the same, as shown in Fig. 6.

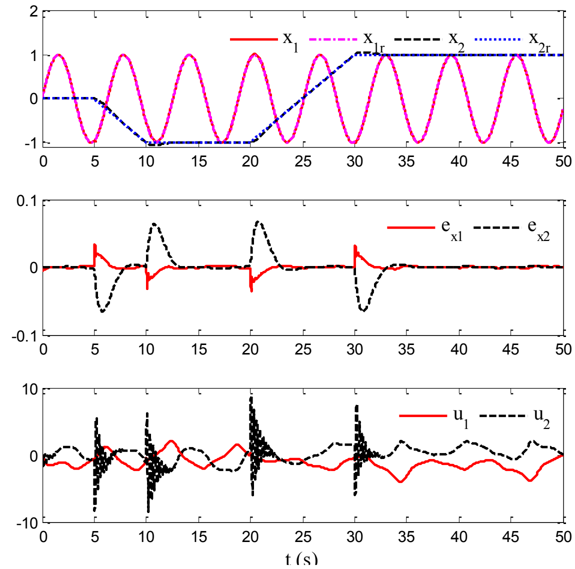

Fig. 6. Simulation Results with $T = 0.01s$.

From Fig. 6, it can be seen that both outputs coincide very well with their references regardless of the disturbances. The tracking errors are less than 0.1. It can be also observed that the maximum tracking errors occur at the turning points of the reference. This is because the ideal input is not continuous at these turning points. Thus the feedforward inputs experience serious chattering at these turning points in order to approach the ideal input. As the input recover to the ideal input, the chattering disappears and the tracking errors converge to a small region around zero.

To explore the impact of the sampling time (or integral gain), a comparison is made with different sampling time as shown in Fig.7 and Fig. 8.

Comparing Fig. 6~8, it can be found that as the sampling time increases, on one hand, the tracking performance degrades as the tracking error becomes bigger; on the other hand, the input chattering is weakened which is good for the actuator. The impact is also summarized in Fig. 9.

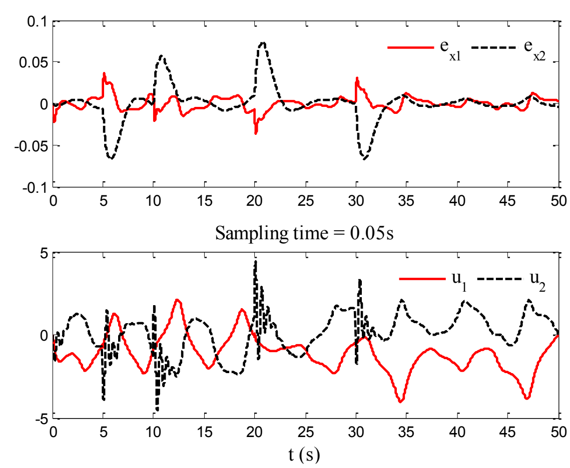

Fig. 7. Simulation Results with $T = 0.05s$.

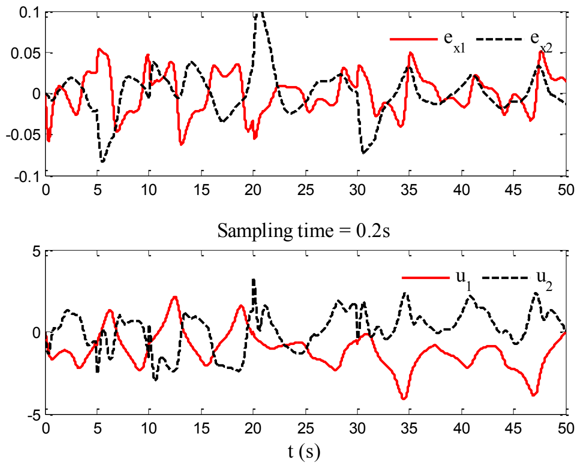

Fig. 8. Simulation Results with $T = 0.2s$.

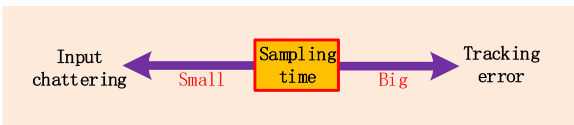

Fig. 9. Impact of Sampling Time.

We can imagine two extreme cases: one is, as $T \to \infty$, the feedforward u_r will not be updated at all and will be identically zero, thus only the feedback part is working and there will be big tracking error; the other is, as $T \to 0$, it indicates that the feedforward u_r will be updated in no time, thus the feedforward tends to be ideal input and there will be nearly zero tracking error which is at the sacrifice of crazy input chattering though. In real applications, a proper sampling time (or integral gain) should be specified to ensure good performance (sampling time not too big) and acceptable input chattering (sampling time not too small).

IV. Open Problem

The basis of this paper is the integral chain system, which has no zero dynamics. For this kind of system, the state references are fully determined by the output references. Thus states errors are available for feedback. However, when it comes to systems with zero dynamics, there will be some internal states which relate to the external states through some dynamic equations. For this kind of system, the internal states references (i.e., ideal internal dynamics [12]) depend on system model. Is it possible to extend UTC to this kind of system is still an open problem.

V. Conclusions

Two formulations of universal tracking controller (UTC) are developed in this paper. One is proportional-integral tracking controller (PITC) which originates from PID control and the other is adaptive feedforward tracking controller (AFTC) which is inspired from the tracking control theory. They are equivalent to each other under certain conditions. UTC gives us an in-depth understanding of tracking control as well as a new perspective to view PID control. We also find that a proper sampling time (or integral gain) can be specified to make a trade-off between tracking performance and input chattering. UTC is a model-free controller and has great learning ability. Due to its excellent tracking ability and disturbance rejection nature, UTC may be applied to various actual control systems to possibly improve the tracking performance in the future.

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